

Compton light pressure and spectral imprint of relic radiation on cosmic electrons

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A fully QED/relativistic theory of light pressure of CMB radiation and Fokker-Planck equation for electron distribution combined with cosmologic relation for CMB temperature, T , yields analytic results for the evolution of the distribution over large span of time and energies. A strong imprint of CMB on electrons transpires *via* formation of “frozen non-equilibrium” state of electrons in current epoch, and possible existence of cutoff and narrow spectral lines as remnants of high- T sources.

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Beginning with its discovery [1], light pressure played an expanding role in astrophysics and cosmology, where of the most current interest is a light pressure by an isotropic Cosmic Microwave Background (CMB) [2] radiation on charged particles resulting in the loss of their momentum/energy [3], *via* inverse Compton scattering [4]. While extremely weak, CMB can also strongly affect high-energy baryons’ fast decay facilitated by the pion production *via* so called Δ -resonance of a baryon and high-energy photons *via* secondary production of virtual pairs, resulting in the existence [5] of an upper limit of the energy of cosmic rays ($\sim 0.5 \times 10^{20} \text{ eV}$). One would expect even stronger and better traced CMB interaction with electrons, especially because of their fundamental nature, whereby the momentum decay of electrons can be treated more thoroughly using the dependence of photon scattering cross-section on photon energy due to the virtual electron-positron pair creation and annihilation described by the QED theory [6].

The CMB light pressure is the only force imposing thermal equilibrium on electrons in the deep space. In this Letter, we show that very interesting effects in such an interaction can be elicited at both low- and high-energy domains. At low-energy, we predicts the formation of a “frozen non-equilibrium” state of electrons as the universe expands (including present epoch), whereby due to its cooling, CMB fails to enforce thermal equilibrium on electrons, thus letting them keep constant temperature forever ($10 - 20 \text{ K}$ depending on cosmological constant). In high-energy domain, where electron energy exceed a so called Compton threshold ($E_C \sim 10^{14} \text{ eV}$), we predict the transformation of initial thermal electron spectra of high-energy sources into narrow lines followed by a cutoff near the Compton threshold. We limit our consideration here only to the momentum decay due to CMB and do not consider other evolution channels (such as e. g. synchrotron radiation due to galactic magnetic fields, secondary effects due to decay of protons, anisotropy fluctuations due to Sachs-Wolfe effect, etc.).

Toolkit. A major tool in our investigation is a light pressure F by a black-body isotropic radiation (in particular, CMB) on an electron. In a recent work [7] we derived a general, fully QED/relativistic formula for the

force F based on Lorentz transformation of an *arbitrary* spectrum $\rho(\epsilon)$ of (dimensionless) photon energies $\epsilon = \hbar\omega/m_0c^2$ (here m_0 is the rest mass of an electron), in a preferred frame, L , where the radiation is isotropic, upon transition from that frames to the frame, R , where the particle is at rest. The theory is valid also for any energy dependence of a cross-section $\sigma(\epsilon)$ of scattering of an ω -photon at a particle. For a particular case of

(1) a black-body (Planck) radiation of an arbitrary temperature T , with its spectral, ρ_{BB} , and total energy, $W_{BB} = W_C \int_0^\infty \epsilon \rho_{BB}(\epsilon) d\epsilon$ densities in the L -frame being

$$\rho_{BB}(\epsilon) d\epsilon = \frac{8\pi\epsilon^2 d\epsilon}{e^{\epsilon/\theta} - 1}; \quad W_{BB} = \frac{8\pi^5}{15} W_C \theta^4 \quad (1)$$

where $\theta = k_B T/m_0c^2$ is a dimensionless temperature (with $T = T_0 \approx 2.725 \text{ K}$, and $\theta = \theta_0 \approx 0.534 \times 10^{-9}$ for present CMB), k_B is the Boltzmann constant, $W_C = m_0c^2/\lambda_C^3$ is a “Compton energy density”, and $\lambda_C = 2\pi\hbar/m_0c$ is the Compton wavelength, and

(2) an electron as a scattering particle, with its cross-section, $\sigma(\epsilon)$, described by Klein-Nishina QED theory [6] accounting for virtual electron-positron pair creation and annihilation in the 1-st order of the fine structure constant $\alpha = e^2/m_0c^2 \approx 1/137$, for an *arbitrary* ϵ , so that $\sigma \approx \sigma_0 = (8\pi/3)r_0^2$ at $\epsilon \ll 1$ is a classical, or Thompson cross-section of electron, where $r_0 = e^2/m_0c^2$ is the classical electron EM-radius, and $\sigma \approx \sigma_0(3/8\epsilon)[\ln(2\epsilon) + 1/2]$ at $\epsilon \gg 1$ in a QED, or Compton domain,

we found a simple and precise analytic approximation [7] for the dimensionless light pressure force $f = Ft_C/m_0c$ in terms of the electron momentum $\mu \equiv p/m_0c$, relativistic factor $\gamma = \sqrt{1 + \mu^2}$, and temperature θ as

$$f(\mu, \theta) = \frac{d\mu}{dt} t_C \approx -\frac{\mu\theta^3}{q} \ln(1 + K_C); \quad K_C = \gamma\theta q \quad (2)$$

where K_C is a “Compton factor”, $q = 10.0$ is a numerical fitting parameter, and t_C is a “Compton time scale”:

$$t_C = 135\lambda_C/64\pi^4\alpha^2c \approx 3.2515 \times 10^{-18} \text{ s}; \quad t_C \propto \hbar^3 \quad (3)$$

Eq. (2) remains true in the entire span of momenta, $\mu \in (0, \mu_{Pl})$, where $\mu_{Pl} = k_B T_{Pl}/m_0c^2 \approx 2.4 \times 10^{22}$ is the highest momentum in the universe related to the

Planck temperature, $T_{Pl} \approx 1.417 \times 10^{32} K$. The Thompson domain corresponds to $K_C \ll 1$ (hence $\theta \ll 1$), with

$$f \approx -\mu\gamma\theta^4, \quad \gamma = \sqrt{1+\mu^2} \quad (4)$$

consistent with a well known result (see e. g. [8]); note that it is still good for relativistic case, $|\mu| \sim \gamma \gg 1$, as long as $|\mu| \ll \theta^{-1}$. The Compton (QED) domain is defined by $K_C \gg 1$, and its threshold, $K_C = 1$, for present CMB corresponds to the energy $E_C \sim 10^{14} eV$.

While Eq. (2) can be directly used to calculate the decay of momentum $\mu(t)$ for any given initial condition (see below), the temporal evolution of electron *distribution* could be found from a Fokker-Planck equation for the diffusion in the momentum space [9]. To that end, we define a distribution function, $g^{(e)}(\mu, t)$ of electrons as the number of electrons per elements of solid angle dO , momentum, $d\mu$, within a unity of coordinate space, and a density number, $\rho^{(e)}(\mu, t) = 4\pi\mu^2 g^{(e)}$, and note that in the expanding space/universe, these functions reflect the distribution within the expanding unity of coordinate space, i. e. in the spatial “unity box” expanding at the same rate as the universe. Assuming then that (a) the electron distribution is isotropic, same as CMB, (b) the total number of electrons, in the unity of momentum space and the expanding unity of coordinate space is approximately invariant, $\int_0^\infty \rho^{(e)} d\mu = inv$, and (c) the thermal equilibrium of a relativistic gas at any $\theta = const$ is due to the Maxwell-Jüttner (MJ) distribution [10],

$$g_{MJ}^{(e)} \propto e^{-\gamma/\theta} [\theta K_2(1/\theta)]^{-1} \quad (5)$$

where K_2 is the modified Bessel function of the second order, with MJ being a relativistic generalization of the Maxwell-Boltzmann (MB) distribution,

$$g_{MB}^{(e)} \propto e^{-\mu^2/2\theta} \theta^{-3/2} \quad (6)$$

we found [7] a Fokker-Planck equation for $g^{(e)}(\mu, t)$, as

$$\frac{\mu^2 \partial[g^{(e)}]}{\partial(t/t_C)} + \frac{\partial}{\partial\mu} \left\{ \mu^2 f(\mu, t) \left[g^{(e)} + \theta(t) \frac{\gamma}{\mu} \frac{\partial g^{(e)}}{\partial\mu} \right] \right\} = 0 \quad (7)$$

In non-relativistic case [$\gamma \approx 1$ in Eq. (4)], it comes to

$$\frac{\mu^2 \partial[g^{(e)}]}{\partial(t/t_C)} = \theta^4(t) \frac{\partial}{\partial\mu} \left\{ \mu^3 \left[g^{(e)} + \frac{\theta(t)}{\mu} \frac{\partial g^{(e)}}{\partial\mu} \right] \right\} \quad (8)$$

Finally, when tackling the dynamics of CMB temperature, T , in the expanding universe, we use the fact that it is directly related to the redshift z as $T(t)/T_0 = 1+z(t)$ (T_0 is a present value), and thus is governed by a standard cosmologic relation [11,12]:

$$dT/dt = -TH_0 \sqrt{\Omega_\Lambda + \Omega_M(T/T_0)^3 + \Omega_R(T/T_0)^4} \quad (9)$$

where H_0 is a present Hubble constant (with $H_0^{-1} \approx 4.414 \times 10^{17} s$ being an approximate age of the universe), Ω 's are the fractions of respective forms of energy in critical energy density (it is a common convention that our

universe is flat, hence $\Omega_\Lambda + \Omega_M + \Omega_R = 1$) with commonly accepted values $\Omega_\Lambda \approx 0.7$ (a vacuum energy density fraction, or cosmological (or *dark energy*) constant, a major contributor to the current rate of the universe expansion), and $\Omega_R \sim 0.85 \times 10^{-4}$ – radiation, or relativistic fraction, dominant at the earlier stage of the universe, and non-relativistic, or “matter” fraction $\Omega_M \approx 0.3$.

Frozen non-equilibrium. How promptly an electron distribution equilibrates with CMB temperature, $\theta(t)$, especially if that one is changing? To address the issue, we compare time scales of both of them. Let us assume that the scale of CMB is the age of universe, t_U , at CMB temperature, θ , i. e. $t_U(\theta) = \int_\infty^\theta d\theta/(d\theta/dt)$. Using Eq. (2), we introduce an electron momentum decay rate, $R = (t_C/\mu)|d\mu/dt|$, and related time scale $t_R = t_C/R$, evaluate $\mu = \mu_{pk}$ at the peak of distribution $\rho^{(e)}(\mu, t)$ for a “current” equilibrium, and then solve the equation $t_U(\theta) = t_R(\theta)$ for a split-point $\theta = \theta_{spl}$ numerically. We found then that with $\Omega_M \gg \Omega_R$, $T_{spl}/T_0 \lesssim 10$, or $\theta_{spl} < 0.5 \times 10^{-8}$, which is also consistent with more detailed calculations, see Fig. 2, and the split occurred at $t \lesssim 0.05 H_0^{-1}$. The main point here is that it falls far within Thompson domain, $\theta_{spl} \ll 1$, so that the electron kinetics could be described by classical Eq. (8).

In the earlier epoch, the thermalization of electrons happened almost instantaneously, so that their distribution is described by Eq. (5) and (6) with the temperature, $\theta_e(t)$ of this distribution following almost exactly the CMB temperature, $\theta(t)$. To investigate what happened after they start diverging near $\theta = \theta_{spl}$, we need to solve Eq. (8) with an initial condition given by MB-distribution (6) at any point $1 \gg \theta \gg \theta_{spl}$. Most luckily, that partial derivative equation happens to have an exponential MB-distribution (6) as an auto-model solution [13], where the temperature θ has to be replaced by an electron temperature, $\theta_e(t)$, as yet unknown function of time, and thus Eq. (8) can be reduced to an ordinary differential equation for $\theta_e(t)$, where the CMB temperature, $\theta(t)$, could still be an arbitrary function of time:

$$d\theta_e/dt = -2\theta^4(t)[\theta_e(t) - \theta(t)]/t_C \quad (10)$$

Eqs. (9) and (10) can now be used to solve the dynamics of both $\theta = k_B T/m_0 c^2$ and $\theta_e = k_B T_e/m_0 c^2$. It suffices, however, to find T_e as function of T ; eliminating the time t by dividing Eq. (10) by (9), we get then a single equation in the phase space of $\Theta = T/T_0$, $\Theta_e = T_e/T_0$ as:

$$d\Theta_e/d\Theta = \chi \Theta^3 (\Theta_e - \Theta) / \sqrt{\Omega_\Lambda + \Omega_M \Theta^3} \quad (11)$$

where we dropped the term $\Omega_R \Theta^4$, which is negligible at $\Theta \lesssim 10^2$, and introduced a “QM+cosmic” parameter

$$\chi = 2(k_B T_0/m_0 c^2)^4 / t_C H_0 \approx 1.21 \times 10^{-2} (\approx 5\alpha/3) \quad (12)$$

which is of unexpectedly “mild” value, as compared to the entries that define it. Note that within known precision of H_0 , χ is well approximated by $5\alpha/3$; it would

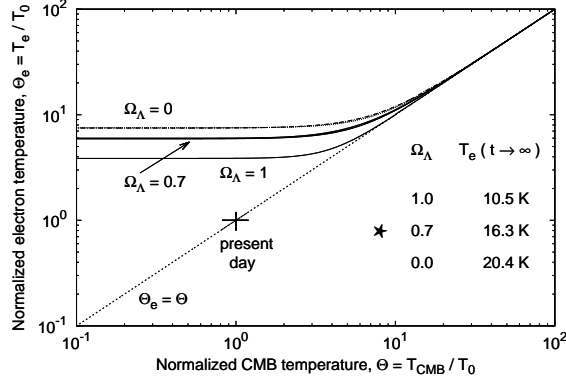


FIG. 1: Normalized temperature of electrons T_e/T_0 , vs that of CMB, T/T_0 , for various values of cosmological constant, Ω_Λ , and asymptotic electron temperatures, T_e , at $t \rightarrow \infty$. A star marks the data for a commonly accepted model [11,12].

be quite surprising and revealing if that is not a chance coincidence. The boundary condition for the solution of Eq. (11) is $(\Theta_e - \Theta) \rightarrow 0$ at $\Theta \rightarrow \infty$.

The results of numerical solutions of Eq. (11) are depicted at Fig. 1. They clearly show that below $\Theta \approx 20$, CMB has “dropped the ball” and cannot enforce thermal equilibrium on cosmic electrons, whose temperature got eventually frozen at some non-equilibrium level ($T_\infty = 16.3\text{K}$ for $\Omega_\Lambda = 0.7$) till “the end of time”. This brings up a new facet to the issue of “heat death” of the universe. [Note, however, that by definition the density number $\rho^{(e)}$ was assigned to a volume expanding with the universe, so that the spacing between electrons increases as $\Theta^{-1}(t)$.] This frozen state is fully developed by the present day, regardless of specific values of Ω ’s in Eqs. (9) or (11). A good analytical approximation for T_∞ and the solution of Eq. (11) for various Ω ’s is found as [14]

$$\Theta_e = \left(\Theta^{5/2} + \Theta_\infty^{5/2} \right)^{2/5} \quad \text{with} \quad (13)$$

$$\Theta_\infty \equiv \frac{T_\infty}{T_0} = \left(\frac{15}{8} \frac{\sqrt{\Omega_M + \delta}}{\chi} \right)^{2/5}; \quad \delta \approx 0.036 \quad (14)$$

Thus conceivable measurements of T_∞ in deep space may offer an alternative way to evaluate $\Omega_\Lambda \approx 1 - \Omega_M$.

Narrow lines and cutoff in cosmic electron spectra? While at the low end of electron spectrum the CMB “withdrew from the game”, it could be expected that, similarly to baryons, it might strongly affect high-energy electron spectrum, albeit due to different mechanism, and do it on a much faster time-scale, so we can even assume $\theta = \text{const} = \theta_0 \ll 1$. To illustrate that, we consider the dynamics of the momentum $\mu(t)$ whose implicit solution for a given θ , is provided by $t(\mu) = t_C \int d\mu/f(\mu, \theta)$ as $f = t_C d\mu/dt$. The integration here can be done numerically [for Eq. (2), the above integral is not evaluated in terms of analytical functions], yet to gain the insights provided by analytical results, it would be nice to have a “good” model function f_M that

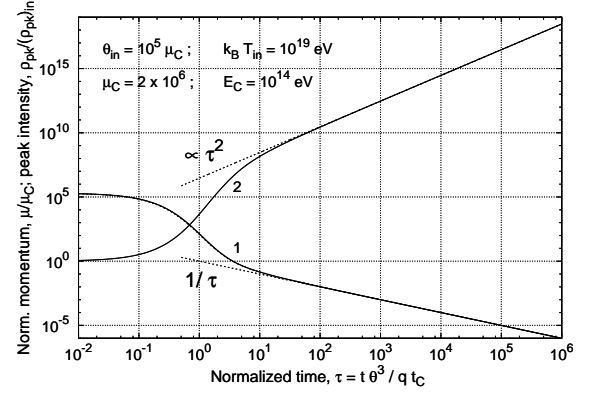


FIG. 2: Normalized momentum, μ/μ_C , and position of the peak of density distribution (curve 1), and the peak intensity of the distribution, $\rho_{pk}/(\rho_{pk})_{in}$, vs normalized time, τ (curve 2). Dashed lines – respective asymptotics for $\tau \gg 1$.

is very close to the one in Eq. (2) in the domain of interest, and at that has (a) an analytical integrability of $\int d\mu/f$, and (b) explicit “reversibility” of resulting functions $t(\mu) \leftrightarrow \mu(\tau)$. In the Thompson domain, $K_C < 1$, Eq. (4) satisfies these conditions and is fully solvable [7]. But to cover both the upper (and largest) part of Thompson domain, $\mu \gg 1$, and at the same time – the entire immensely larger, Compton domain, $K_C > 1$, another greatly useful interpolation model is found as

$$f_M(\mu, \theta) = -(\theta^3/q)y(1 + \ln y) \left\{ 1 + [\ln(1 + \ln y)]^{-2} \right\}^{-1} \quad (15)$$

where $y = 1 + \mu/\mu_C$ with $\mu_C = 1/q\theta \gg 1$; $\mu > 1$. For $\theta = \theta_0$, we have $|f - f_M|/f < 0.01$, for any $\mu > 7$. At $1 \ll \mu \ll \mu_C$, Eq. (15) yields $f_M \approx -\theta^4 \mu^2$, which is consistent with Eq. (3) at $\mu \approx \gamma \gg 1$, i. e. only for relativistic case. Yet this is more than enough if $\theta \ll 1$ by insuring that momentum decay can be continually traced from far Compton to low Thompson domains. If necessary, the calculations for electron decay at $\mu \ll \mu_C$ can be picked up by Eq. (4). Thus Eqs. (4), (15) together smoothly cover the entire span $\mu \in (0, \mu_{Pl})$, as their areas of validity overlap by orders of magnitude in μ if $\theta \ll 1$. The momentum decay from initial $\mu = \mu_{in}$ at $\tau = 0$, via integration $d\mu/dt = f_M/t_C$, with $\theta = \text{const}$ is

$$\frac{\mu(\tau)}{\mu_C} = \exp \left\{ \exp \left[\sqrt{\frac{(\tau_0 + \tau)^2}{4} + 1} - \frac{\tau_0 + \tau}{2} \right] - 1 \right\} - 1 \quad (16)$$

where $\tau = (\theta^3/q)t/t_C$ and

$$\tau_0 = s^{-1} - s \quad \text{with} \quad s = \ln[1 + \ln(1 + \mu_{in}/\mu_C)] \quad (17)$$

For $K_C \approx \mu_{in}/\mu_C = 2 \times 10^5$ (or initial energy $E_{in} \sim 2 \times 10^{19} \text{ eV}$ slightly below the highest particle energy $\sim 5 \times 10^{19} \text{ eV}$, observed in cosmic rays [15]), $\mu(\tau)$ is depicted in Fig. 2, curve 1. The time $\int_{\mu_C}^{\mu} d\mu/|f_M|$ for an electron to lose about 2×10^5 of its momentum during

the “Compton phase” $\mu_{in} \rightarrow \mu_C$ is $\Delta\tau_C \sim 2.57$, hence $(\Delta t)_C \sim 5.3 \times 10^{11} s$, which is by 6 orders of magnitude shorter than the age of universe (and thus justifies our assumption of $\theta \approx const$), whereas immediately after that, within the same period, μ loses much less than a factor of magnitude. (For the sources with $E_{in} < 5 \times 10^{19} eV$, this time will be even shorter.) Following that, μ decays even slower as $\propto 1/t$. The explanation of this dramatic difference (see e. g. [3]) is that while the cross-section of photon scattering by electrons is slowly receding with energy, each act of inverse Compton scattering gets more “quantum efficient” at $\mu \gg \mu_C$ [7], whereby a low-energy photon scattered from a high-energy electron gets a huge boost by accruing up to $\mu_0 - \mu_C$ momentum. As μ keeps decaying from μ_C down to a relativistic threshold, $\mu = 1$, its dynamics slows down tremendously, which eventually leads to a frozen non-equilibrium at lower μ .

From the momentum decay of single electron it is obvious that the high-energy electron spectrum could be greatly affected by CMB, which make it of special interest to look into the evolution of electron spectra at the energies far exceeding that of equilibrium, or even Compton threshold, $E_C \sim 10^{14} eV$. At that, the system remains far from the equilibrium during the evolution, and the last term in a Fokker-Planck Eq. (7) can be omitted since $\theta_0 \ll \theta_{in}$, so that in terms of number density $\rho^{(e)} \propto \mu^2 g^{(e)}$ it can be reduced to

$$t_C \partial \rho^{(e)} / \partial t + \partial [f \rho^{(e)}] / \partial \mu = 0 \quad (18)$$

which is essentially a continuity-like equation. Again, it is fully integrable, and its general solution is

$$\rho^{(e)} = \Phi(\xi - t) / f(\mu), \quad \text{with} \quad \xi = t_C \int d\mu / f \quad (19)$$

where $\Phi(x)$ is an arbitrary function of x defined here by initial conditions, e. g. a MJ-distribution with $\theta_{in} \gg 1$. A resulting analytic solution for $\rho^{(e)}(\mu, \tau)$ with $f = f_M$, Eq. (15), for $\rho^{(e)}$ vs μ for various $\tau = (\theta^3/q)t/t_C$ is plotted in Fig. 3 for initial temperature, $\theta_{in} = 10^5 \mu_C$ or $k_B T_{in} = 10^{19} eV$. A curve at $\tau = 0$ depicts an initial MJ-distribution, $\rho_{in}^{(e)}(\mu) \propto \mu^2 e^{-\gamma/\theta_{in}}$, which peaks at $\mu = 2\theta_{in}$, i. e. $E_{in} = 2 \times 10^{19} eV$, same as for a single electron example. A transient peak at $\mu = \mu_{pk}$ moves fast in the beginning, but slows down tremendously as it reaches $\mu_C(\theta)$. Its motion coincides with the timeline of a single electron with $\mu_{in} = 2\theta_{in}$, see curve 1 in Fig. 2, whereas its intensity $\rho_{pk}(\tau)$, curve 2 in Fig. 2, goes up orders of magnitude higher than that of the initial MJ-distribution; at $\mu < \mu_C$, $\rho_{pk} \propto \tau^2$. Its width narrows down respectively, $\Delta\mu/\mu_{pk} \propto \tau^{-2}$ so that for e. g. $\mu = 20$ ($E = 10 MeV$), it reaches $\Delta E \sim 2 KeV$ i. e. $\Delta\mu/\mu_{pk} \sim 2 \times 10^{-4}$, compared to the initial relative width $\Delta\mu_{in}/\mu_{in} \sim 1.4$. Notice that even before strong line-narrowing, there is a sharp cutoff at the upper part of the spectrum. This collapse and cutoff are due to a “pile-up” effect, whereby a leading downward front moves

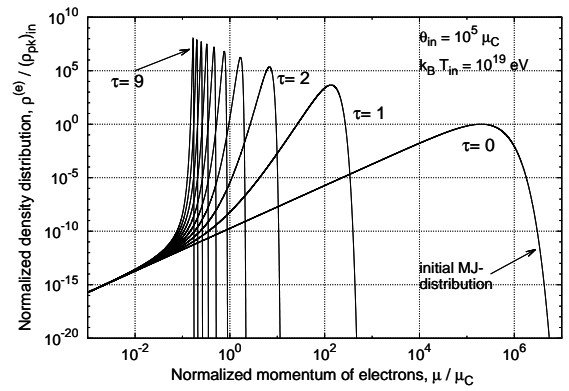


FIG. 3: Evolution of normalized density distribution of electrons, $\rho^{(e)}(\mu)/(\rho_0^{(e)})_{pk}$ in normalized time, τ , beginning with the initial, Maxwell-Jüttner (MJ) distribution at $k_B T_{in} = 10^{19} eV$, from $\tau = 0$ to $\tau = 9$ ($\Delta\tau = 1 \rightarrow \Delta t \approx 2 \times 10^{11} s$).

slower than a trailing one, resulting in the line squeezing; it is reminiscent of a shock precursor formation in astrophysics [16] and Coulomb explosion [17].

These narrow lines would indicate signals from very far and hot sources; most likely they will be very weak. Observation efforts may necessitate the development of high-resolution spectral techniques for their detection. More detailed study involving anisotropic Fokker-Planck equations [expanding Eqs. (7) and (18) used here to demonstrate the effect] are to be used for data analysis. The averaging over many sources is expected however to be isotropic, although the observed line might be broaden up similarly to the inhomogeneous line broadening in laser physics [18]. However, the major feature common to all of those individual spectra is a sharp cutoff in each one of them beyond the lines in a close vicinity of Compton threshold, $E_C \sim 10^{14} eV$, which might be another target to search for in the background spectral noise around E_C .

In conclusion, we showed that a greatly diminished light pressure on electrons by relic radiation and ensuing low rate of their energy decay should result in the formation of frozen non-equilibrium state of cosmic electrons with the temperature of 10 – 20 K as the universe expands long before the current epoch. By exploring the evolution of spectra of high- T sources, we also predicted their implosion into very narrow lines and cutoff formation due to pile-up effect.

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- [13] In fact, Eq. (8) can be solved the same way for an *arbitrary* initial function $g_{in}^{(e)}(\mu)$, by decomposing it into exponential components *via* Laplace transformation, and looking for the solutions of Eq. (10) with initial magnitude θ_{ein} different for each of the components.
- [14] If one of Ω ’s in Eq. (11) is unity (hence the other is zero), Eq. (11) can be integrated in close form in terms of Γ -function. In particular, for $\Omega_\Lambda = 1$ (and thus the lowest possible temperature T_∞), its exact solution is $\Theta_e = \Theta + (1/4)(4/\chi)^{1/4} \exp(\chi\Theta^4/4)\Gamma(1/4, \chi\Theta^4/4)$, where $\Gamma(a, x)$ is an upper incomplete Γ -function, and $T_\infty/T_0 = (64\chi)^{-1/4} \Gamma(1/4) \approx 3.86$, where $\Gamma(a) = \Gamma(a, 0)$ is a complete Γ -function. Thus $T_\infty \approx 10.52K$, which is consistent with the numeric calculations, Fig. 1, and Eq. (14).
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